26. Examples

The `Dash`, `Dashto` and `Mono` cell types in some of these examples use the `arrsy10` font: See §14 and `\arrsy` in §25. Also, `\bb` invokes the bbm family of fonts.

```
$$
\grid=.75cm
\Diagram
A'          &       & \rTo ^{f'} &       & B'                          \\
            & \rdTo <a \dh{2pt}& &       &         & \rdTo _b          \\
\dTo <{g'} &       & A          &       & \dTo <{h'} :{.25}
                                          \rTo ^f :{.25} \br
                                                  &          & B       \\
                                                                       \\
C'          &       & \rTo ^{k'} :{.75} \dTo <g :{.25} \br
                                  &       & D'      &          & \dTo >h \\
            & \rdTo _c \dh{.5pt}&&       &         & \rdTo _d          \\
            &       & C          &       & \rTo ^k &          & D       \\
\endDiagram
$$
```

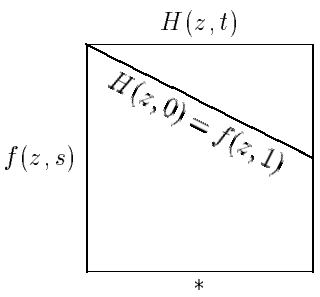
$$\begin{array}{ccccc}
A' & & \xrightarrow{f'} & & B' \\
& \overset{a}{\searrow} & & \overset{b}{\searrow} & \\
g' \downarrow & & A & \xrightarrow{f} & B \\
& & \downarrow{\scriptstyle g} & & \downarrow{\scriptstyle h} \\
C' & & \xrightarrow{k'} & & D' \\
& \underset{c}{\searrow} & & \underset{d}{\searrow} & \\
& & C & \xrightarrow{k} & D
\end{array}$$

G. Bredon, *Introduction to Compact Transformation Groups*, page 75.

```
$$
\Dg
F\times K\times U     & \rTo ^{\rm proj}     &           & K\times U        \\
\dTo                  &                      &           & \dTo >{\rm proj} \\
F\times_K (K\times U) & {} \approx F\times U & \rTo ^{\rm proj}& U          \\
\endDg
$$
```

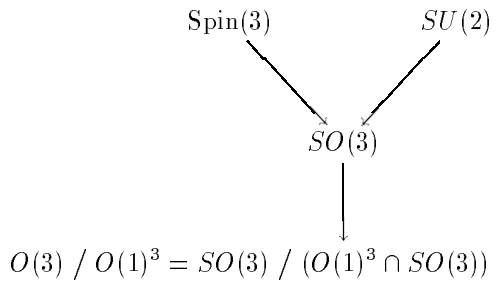
$$\begin{array}{ccc}
F \times K \times U & \xrightarrow{\text{proj}} & K \times U \\
\downarrow & & \downarrow{\scriptstyle \text{proj}} \\
F \times_K (K \times U) \approx F \times U & \xrightarrow{\text{proj}} & U
\end{array}$$

```
$$
\joined \let\labelstyle\textstyle
\Graph{3cm,2}{3cm,4}
\Line (0,0) (0,4) <{f(z,s)}
\Line (0,4) (2,4) ^{H(z,t)}
\Line (0,0) (2,0) _{*}
\Line (2,0) (2,4)
\Line (0,4) (2,2) _{\;\;H(z,0) = f(z,1)} \ro
\endGraph
\qquad\qquad
H'(z,t,s) =
\cases{f(z,2s/(2-t))& if $0\le 2s\le 2 - t$\cr
       \cr
       H(z,2s - 2 + t)& if $2 - t\le 2s\le 2$\cr}
$$
```



$$H'(z,t,s) = \begin{cases} f(z, 2s/(2-t)) & \text{if } 0 \le 2s \le 2-t \\ H(z, 2s-2+t) & \text{if } 2-t \le 2s \le 2 \end{cases}$$

```
$$
\def\mod{\mathrel{\big/}}
\Dg
{\rm Spin}(3) &        &       &       & SU(2)       \\
              & \rdTo  &       & \ldTo               \\
              &        & SO(3)                       \\
              &        & \dTo                        \\
& O(3)\mod O(1)^3 = {} & SO(3)\mod(O(1)^3\cap SO(3)) \\
\endDg
$$
```



S. Mac Lane, *Categories for the Working Mathematician*, page 109.

```
$$
\def\cod{{\rm cod}\,} \def\dom{{\rm dom}\,}
\cellwidth+=2mm
\Dg
& F & _{\cod u} & \rEq       & F & _{\cod u} &     &              & F_i                \\
& \uTo >{p_u} & &            & \uTo &        &     & \ruTo <{p_i} \tx{-5pt} &
                                                                  \uDashto >{\mu_i} \\
{\mit\Pi}_{u:j\to k}F_k = {\mit\Pi}_u &
  F & _{\cod u} & \lTo ^f \up{3pt} \lTo _g \up{-3pt}
                              & {\mit\Pi}_i & & F_i & \lDashto ^e & d                \\
& \dTo >{p_u} & &            & \dTo >{p_{\dom u}}                                     \\
& F & _{\cod u} & \lTo ^{Fu} & F & _{\dom u}                                          \\
\endDg
$$
```

$$
\begin{array}{ccccc}
F_{\cod u} & = & F_{\cod u} & & F_i \\
\uparrow{\scriptstyle p_u} & & \uparrow & \nearrow{\scriptstyle p_i} & \uparrow{\scriptstyle \mu_i} \\
\Pi_{u:j\to k}F_k = \Pi_u F_{\cod u} & \underset{g}{\overset{f}{\longleftarrow}} & \Pi_i F_i & \xleftarrow{e} & d \\
\downarrow{\scriptstyle p_u} & & \downarrow{\scriptstyle p_{\dom u}} & & \\
F_{\cod u} & \xleftarrow{Fu} & F_{\dom u} & &
\end{array}
$$

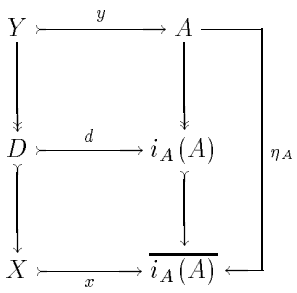
S. Mac Lane, *Categories for the Working Mathematician*, page 117.

```
$$
\Dg
u           & \rTo ^{e_1}   & v       & & \rTo \up{3pt} ^f \rTo \up{-3pt} _g & d    \\
\uDashto >s &               & \dTo >e &                   &                  & \uTo \\
w           & \rTo ^{ee_1s} & w       & {} = {\mit\Pi}k_i & \rTo             & k_i  \\
\endDg
$$
```

$$
\begin{array}{ccccc}
u & \xrightarrow{e_1} & v & \underset{g}{\overset{f}{\rightrightarrows}} & d \\
\uparrow{\scriptstyle s} & & \downarrow{\scriptstyle e} & & \uparrow \\
w & \xrightarrow{ee_1s} & w = \Pi k_i & \longrightarrow & k_i
\end{array}
$$

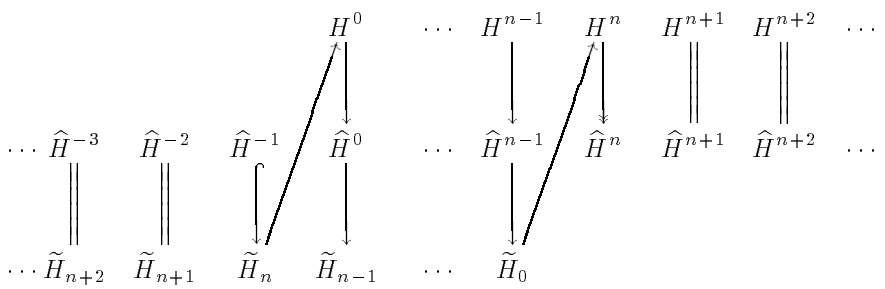
S. Mac Lane and I. Moerdijk, *Sheaves In Geometry and Logic*, page 232.

```
$$
\cellwidth+=3mm
\Dg
Y      & \rMono ^y & A                 & \rLine \jh & \stop                \\
\dEpi  &           & \dEpi             &            & \mx{-1cm}            \\
D      & \rMono ^d & i_A(A)            &            & \dLine >{\eta_A} \jn \\
\dMono &           & \dMono                                              \\
X      & \rMono _x & \overline{i_A(A)} & \lTo \jt   & \stop                \\
\endDg
$$
```



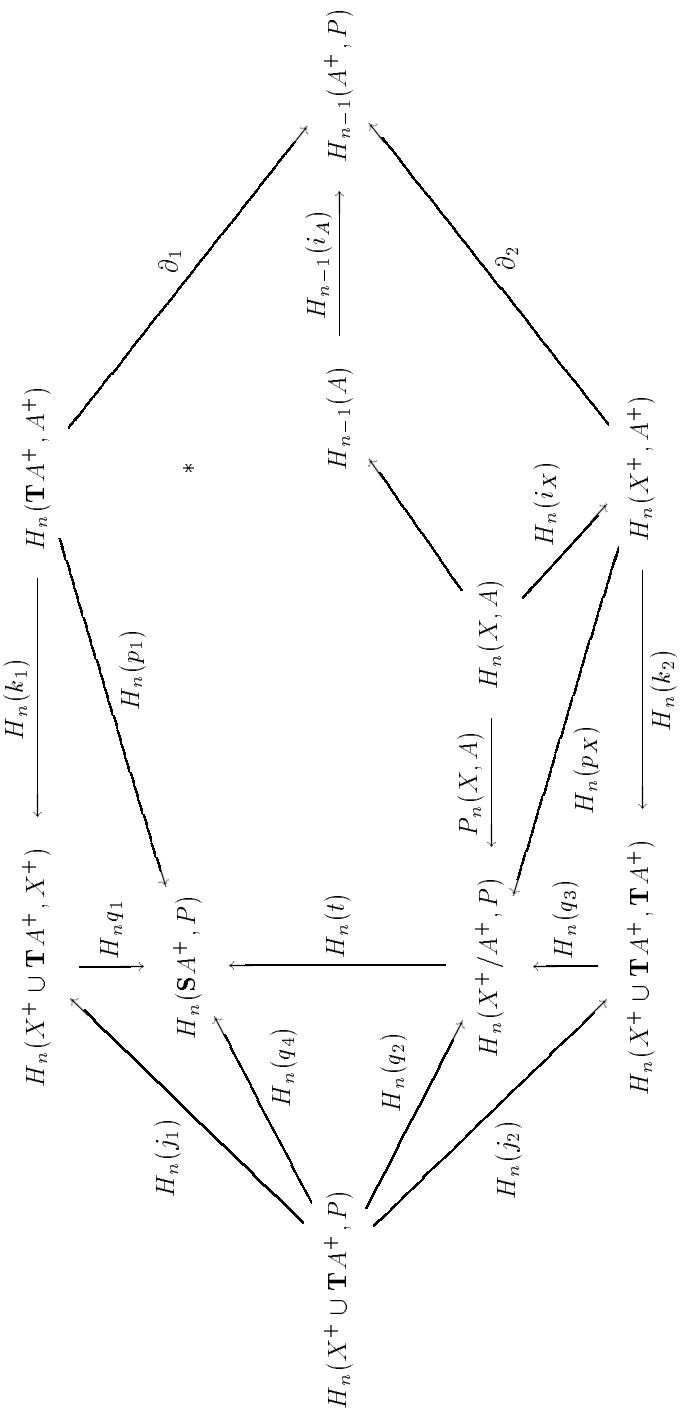
K. S. Brown, *Cohomology of Groups*, page 280.

```
$$
\let\^\widehat \let\~\widetilde
\flexible \columndist=1cm
\xgrid+=2mm \ygrid+=-2mm
\Diagram
        &         &         &        &H^0 \bTo(-1,-4) \hx{-2pt}
                                          &\cdots  &H^{n-1}  &H^n  &H^{n+1}  &H^{n+2} &\cdots \\
\mx{5mm}&         &         &        &\dTo   &        &\dTo     &\dEpi&\dBar    &\dBar &\mx{-2mm}\\
\cdots  &\^H^{-3} &\^H^{-2} &\^H^{-1}&\^H^0  &\cdots  &\^H^{n-1}&\^H^n&\^H^{n+1}&\^H^{n+2}&\cdots\\
        &\dBar    &\dBar    &\dInto  &\dTo   &        &\dTo \dx{-2mm}                              \\
\cdots  &\~H_{n+2}&\~H_{n+1}&\~H_n   &\~H_{n-1}&\cdots&\~H_0 \aTo(1,4) \hx{-2pt}                  \\
\endDiagram
$$
```



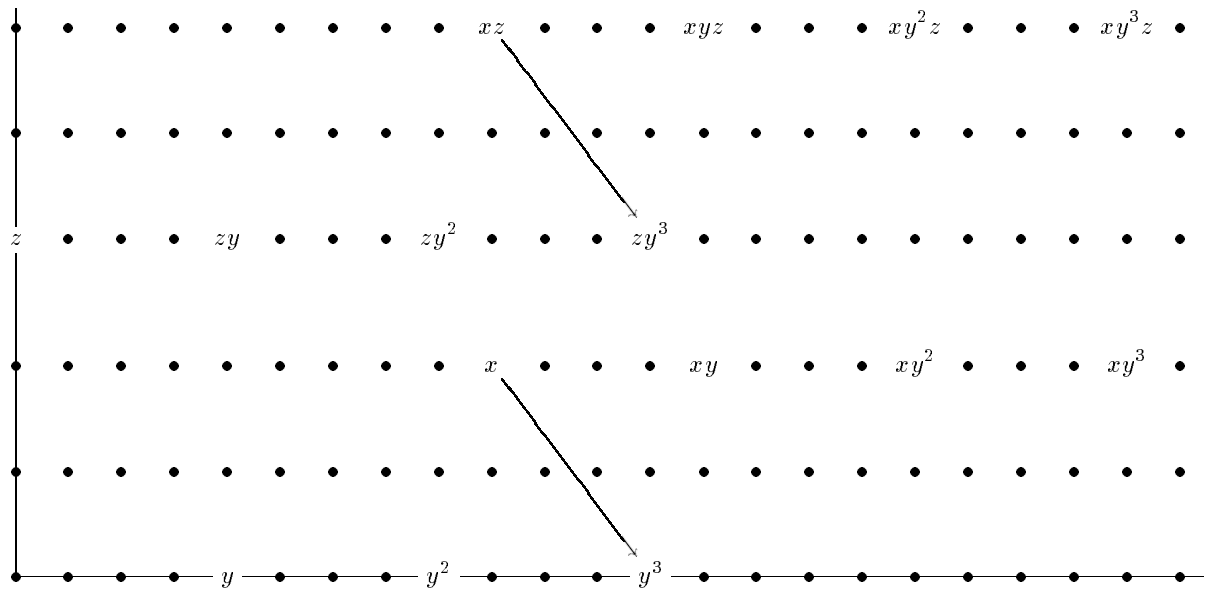
George W. Whitehead, *Elements of Homotopy Theory*, page 592.

```
$$
\def\T{{\bf T}} \def\S{{\bf S}}
\landscape \let\labelstyle\textstyle
\cellpush=10pt \xscale=2.2
\Diagram
       &                  & H_n(X^+\cup\T A^+, X^+) &
   \lTo ^{H_n(k_1)} &        & H_n(\T A^+,A^+)
                              \aTo (-3,-2) _{H_n(p_1)}
                              \aTo (3,-4) ^{\partial_1}                  \\
       &                  & \dTo >{H_n{q_1}}                           \\
       &                  & H_n(\S A^+,P)           &
                     &        & * \dx{.7}                                \\
       & \ruTo _{H_n(q_4)} \tx{15pt}
                          & \uTo >{H_n(t)}                             \\
H_n(X^+\cup\T A^+,P) \aTo (2,4)  ^{H_n(j_1)} \tx{15pt}
                       \aTo (2,-4) _{H_n(j_2)} \tx{15pt}
       &   &   &    &        &   & H_{n-1}(A) &
                                    \rTo ^{H_{n-1}(i_A)} & H_{n-1}(A^+,P) \\
       & \rdTo ^{H_n(q_2)} \tx{15pt}
                          &                          & & & \ruTo           \\
       &                  & H_n(X^+/A^+,P)          &
  \lTo ^{P_n(X,A)}  & H_n(X,A)
                        \aTo (1,-2) >{H_n(i_X)}                          \\
       &                  & \uTo >{H_n(q_3)}                           \\
       &                  & H_n(X^+\cup\T A^+,\T A^+)&
  \lTo _{H_n(k_2)} & & H_n(X^+,A^+)
                         \aTo (3,4) _{\partial_2}
                         \aTo (-3,2) _{H_n(p_X)}                         \\
\endDiagram
$$
```


$H_n(X^+ \cup \mathbf{T}A^+, P)$
$H_n(j_1)$
$H_n(X^+ \cup \mathbf{T}A^+, X^+)$
$H_n(k_1)$
$H_n(\mathbf{T}A^+, A^+)$
∂_1
$H_{n-1}(A^+, P)$
$H_{n-1}(i_A)$
$H_{n-1}(A)$
$H_n(q_4)$
$H_n(\mathbf{S}A^+, P)$
$H_n q_1$
$H_n(p_1)$
$*$
$H_n(t)$
$H_n(q_2)$
$H_n(X^+/A^+, P)$
$P_n(X, A)$
$H_n(X, A)$
$H_n(i_X)$
$H_n(j_2)$
$H_n(q_3)$
$H_n(p_X)$
$H_n(X^+ \cup \mathbf{T}A^+, \mathbf{T}A^+)$
$H_n(k_2)$
$H_n(X^+, A^+)$
∂_2

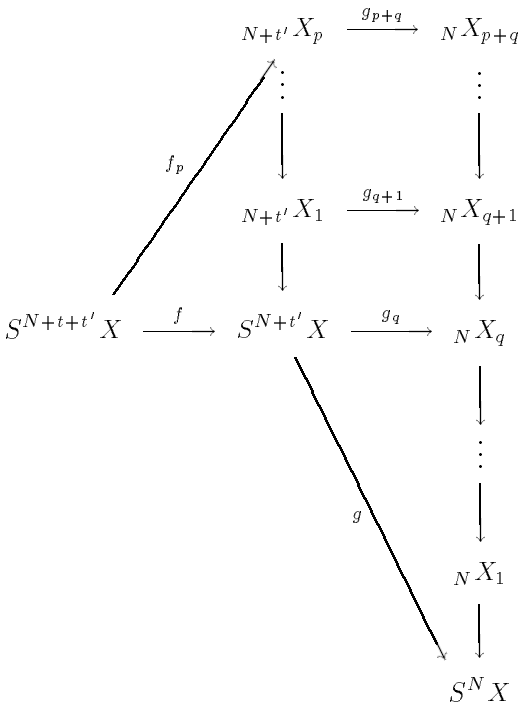
J. McCleary, *A User's to Spectral Sequences*, page 13.

```
$$
\dotted
\grid=7mm
\yscale=2
\Diagram
&&&&&&&&&xz&&&&xyz&&&&xy^2z&&&&xy^3z&\\
\\
z&&&&zy&&&&zy^2&&&&zy^3\\
\dy{-.2}
&&&&&&&&&x&&&&xy&&&&xy^2&&&&xy^3\\
\\
&&&&y&&&&y^2&&&&y^3\\
\Modify
\Line (0,0) (4,0) \dt{1pt}
\Line (4,0) (8,0)
\Line (8,0) (12,0)
\Line (12,0) (22.5,0)
\Line (0,0) (0,3) \dt{1pt}
\Line (0,3) (0,5.2)
\To (9,2) (12,0)
\To (9,5) (12,3)
\endDiagram
$$
```



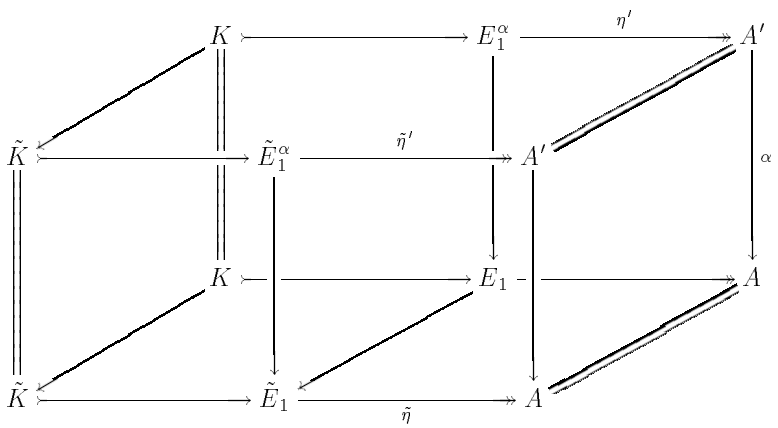
J. McCleary, *A User's to Spectral Sequences*, page 346.

```
$$
\cellpush+=5pt
\cellwidth+=2mm
\Dg
&&_{N+t'}X_p  & \rTo ^{g_{p+q}} & _N X_{p+q}  \\
&& \dFillcell{\;\cdots\;\rightarrowfill} &
                                & \dFillcell{\;\cdots\;\rightarrowfill} \\
                                                \\
&& _{N+t'}X_1 & \rTo ^{g_{q+1}} & _N X_{q+1}  \\
&& \dTo       &                 & \dTo        \\
S^{N+t+t'}X \aTo (2,5) ^{f_p} \tx{.7} \hx{5pt} & \rTo ^f &
   S^{N+t'}X  & \rTo ^{g_q}     & _N X_q      \\
                                                \\
&&            &                 & \dFillcell{\rightarrowfill\;\cdots\;\rightarrowfill} \\
                                                \\
&&            &                 & _N X_1      \\
&&            &                 & \dTo        \\
&&            &                 & S^N X \bTo (-2,6) <g \hx{.2} \\
\endDg
$$
```



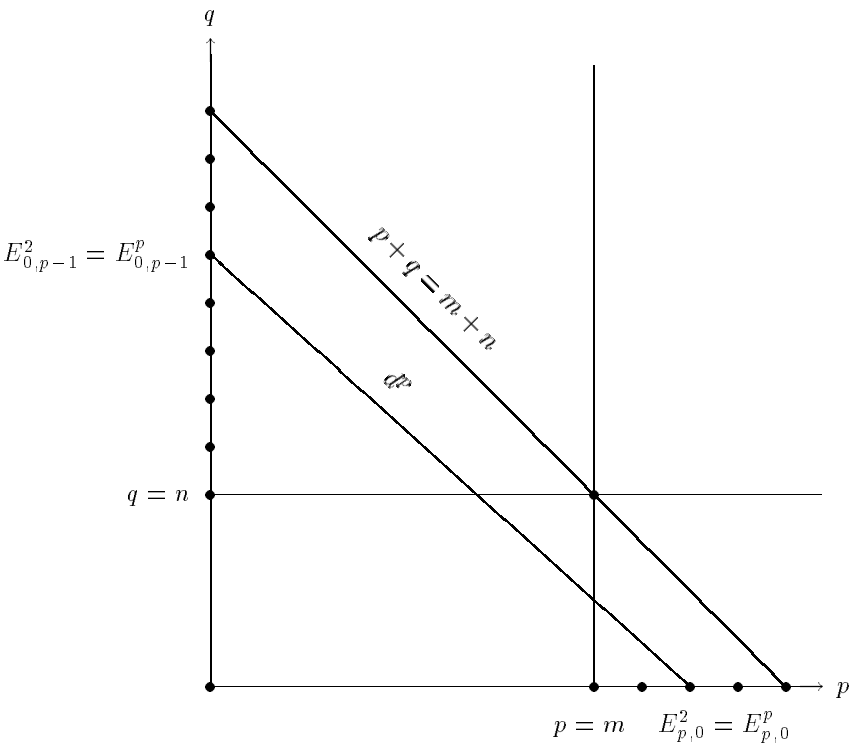
P. J. Hilton and U. Stammbach, *A Course in Homological Algebra*, page 149.

```
$$
\let~\tilde
\cellwidth+=17mm \columndist+=12mm
\gravitateleft
\Dg
  &      & K             & \rMono     && E_1^\alpha& \rEpi^{\eta'}&& A'      \\
  & \ldTo&               &            &&           &      & \ruBar          \\
~K&      & \dBar\rMono\br& ~E_1^\alpha&& \dTo\rEpi^{~\eta'}\br&
                                                      A'                     \\
\dBar&   &               &            &&           &      &    & \dTo>\alpha \\
  &      & K             & \rMono\dTo\br&& E_1\ax{2mm}& \rEpi\dTo\br&& A     \\
  & \ldTo&               &      & \ldTo&           &      & \ldBar          \\
~K&      & \rMono        & ~E_1 & \rEpi_{~\eta}&   & A                      \\
\endDg
$$
```

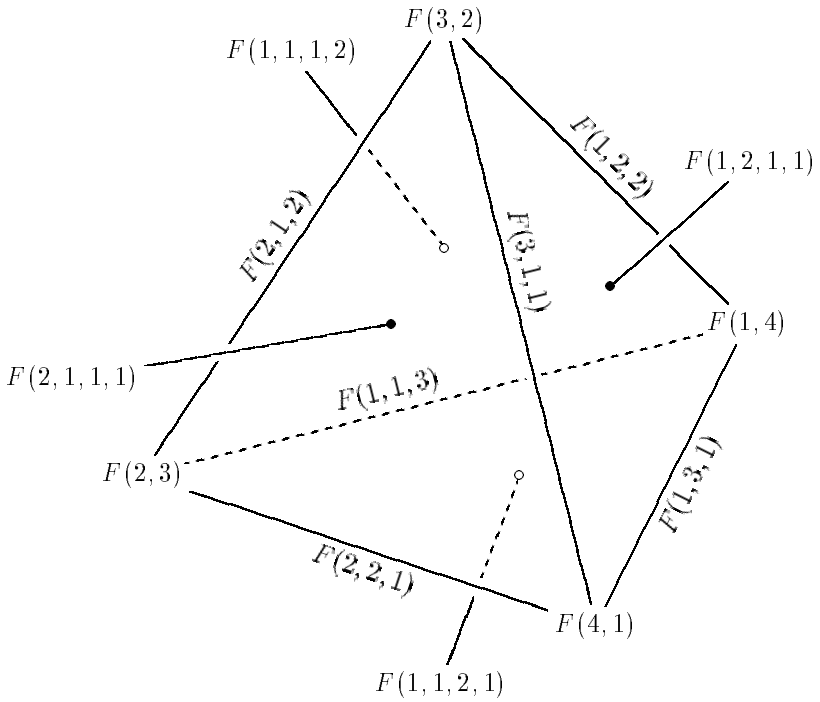


George W. Whitehead, *Elements of Homotopy Theory*, page 650.

```
$$
\cellpush=4pt
\let\labelstyle\textstyle
\lpad=1in \bpad=1cm
\Graph{4in,4}{3.5in,1in}
\To (0,0) (3.3,0)    \hd{p}
\To (0,0) (0,3.5)    \hd{q}
\Line (2,0) (2,3.25) \Line (0,1) (3.2,1) \Dot (2,1)
\Dot (0,0)
%
\Dot (2,0) \Dot (2.25,0) \Dot (2.5,0) \Dot (2.75,0) \Dot (3,0)
%
\Dot (0,1) \Dot (0,1.25) \Dot (0,1.5) \Dot (0,1.75)
\Dot (0,2) \Dot (0,2.25) \Dot (0,2.5) \Dot (0,2.75) \Dot (0,3)
%
\Line (0,2.25) (2.5,0)  ^{d^p} :{.35,5pt} \ro
\Line (0,3) (3,0)  ^{p + q = m + n} :{.35,5pt} \ro
%
\Math{q = n} (-.1,1) *1     \Math{E_{0,p-1}^2 = E_{0,p-1}^p} (-.1,2.25) *1
\Math{p = m} (2,-.2) *{.55} \Math{E_{p,0}^2 = E_{p,0}^p} (2.75,-.2) *{.6}
\endGraph
$$
```



```
$$
\rotatedlabels \let\labelstyle\textstyle \scale=2
\Diagram
 & & F(3,2) \aLine (1,-4) >{F(3,1,1)} :{.4} \br  \\
 & &        & \rdLine >{F(1,2,2)} \pt{right,.75} \\
 & &        &         & F(1,4)                   \\
F(2,3) \aDash (4,1) ^{F(1,1,3)} :{.4} \pp        \\
 & &        & F(4,1) \aLine (1,2) >{F(1,3,1)}    \\
\Modify
\Pt{top} (2,4) \Pt{b1} (0,1) \Pt{b2} (3,0)
\Pt{LeftF} (1.65,2) \Pt{BackF} (2,2.5)
\Pt{BottomF} (2.5,1) \Pt{RightF} (3.1,2.25)
\Nul  \pt{b1} \pt{b2}              \pt{bottom,.8}
\Nul  \pt{BottomF} \pt{bottom}     \pt{BottomL,2}
\Dash \pt{BottomF} \pt{bottom}     \tl\circ \dt{3pt}
\Line \pt{bottom} \pt{BottomL}     \hd{F(1,1,2,1)}
\Line \pt{b1} \pt{b2} _{F(2,2,1)}  \br
\Nul  \pt{b1} \pt{top}             \pt{left,.25} \pt{back,.75}
\Nul  \pt{BackF} \pt{back}         \pt{BackL,2}
\Nul  \pt{LeftF} \pt{left}         \pt{LeftL,2}
\Dash \pt{BackF} \pt{back}         \tl\circ \dt{3pt}
\Line \pt{back} \pt{BackL}         \hd{F(1,1,1,2)}
\Line \pt{b1} \pt{top} <{F(2,1,2)} \br
\Line \pt{LeftF} \pt{LeftL}        \tl\bullet \hd{F(2,1,1,1)} \jt \br
\Nul  \pt{RightF} \pt{right}       \pt{RightL,3}
\Line \pt{RightF} \pt{RightL}      \tl\bullet \hd{F(1,2,1,1)} \jt \br
\endDiagram
$$
```



```
$$
\def\y#1#2{Y(#1,#2)} \def\RP{{\bb R}P}
\gravitateright
\Dg
&&&\RP^1 = {}& \y11 & \rInto & \y21 & \rInto & \y31 & \rInto & \y41 & \rInto & \cdots \\
&&&          &      &        & \dTo &        & \dTo &        & \dTo &                  \\
&&\RP^2 = {}&&      &        & \y12 & \rInto & \y22 & \rInto & \y32 & \rInto & \cdots \\
&&          &&      &        &      &        & \dTo &        & \dTo &                  \\
&\RP^3 = {}&&&      &        &      &        & \y13 & \rInto & \y23 & \rInto & \cdots \\
&          &&&      &        &      &        &      &        & \dTo &                  \\
\RP^4 = {}&&&&      &        &      &        &      &        & \y14 & \rInto & \cdots \\
\endDg
$$
```

$$
\begin{array}{ccccccccc}
\mathbb{R}P^1 = Y(1,1) & \hookrightarrow & Y(2,1) & \hookrightarrow & Y(3,1) & \hookrightarrow & Y(4,1) & \hookrightarrow & \cdots \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
 & \mathbb{R}P^2 = Y(1,2) & \hookrightarrow & Y(2,2) & \hookrightarrow & Y(3,2) & \hookrightarrow & \cdots & \\
 & & & \downarrow & & \downarrow & & & \\
 & & \mathbb{R}P^3 = Y(1,3) & \hookrightarrow & Y(2,3) & \hookrightarrow & \cdots & & \\
 & & & & \downarrow & & & & \\
 & & & & \mathbb{R}P^4 = Y(1,4) & \hookrightarrow & \cdots & &
\end{array}
$$

```
$$
\def\M{{\cal M}\dot} \def\inv{^{-1}} \def\orlz#1{\langle#1\rangle}
\let\labelstyle\textstyle
\Dg
[q(g,b),t] \aNul (1,-1) \aMapsto (2,-4) &&& \rMapsto &&&
                 q(g,\phi(b,t)) \aNul (-1,-1) \aMapsto (-2,-4) \mx{12pt} \\
       & \M R_s &       & \rTo    &        & \beta\inv(W_s)   \\
       &        & \rdTo &         & \ldTo >{R_s}              \\
       &        &       & \beta\inv\orlz{s}                   \\
       &  & q(g,r_s(b)) & {} = {} & q(g,r_s(\phi(b,t)))                 \\
\endDg
$$
```

$$
\begin{array}{ccccc}
[q(g,b),t] & & \longmapsto & & q(g,\phi(b,t)) \\
 & \searrow & & \swarrow & \\
 & & \dot{\mathcal{M}}R_s & \longrightarrow & \beta^{-1}(W_s) \\
 & & & \searrow^{R_s} & \\
 & & & \beta^{-1}\langle s\rangle & \\
 & \searrow & & \swarrow & \\
 & & q(g,r_s(b)) = q(g,r_s(\phi(b,t))) & &
\end{array}
$$