The Atomic Theory as Applied To Gases, with Some Experiments on the Viscosity of Air

by

Silas W. Holman

Submitted to the Department of Physics
in partial fulfillment of the requirements for the degree of

BACHELOR OF SCIENCE IN PHYSICS

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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ABSTRACT

The developments of the "kinetic theory" of gases made within the last ten years have enabled it to account satisfactorily for many of the laws of gases. The mathematical deductions of Clausius, Maxwell and others, based upon the hypothesis of a gas composed of molecules acting upon each other at impact like perfectly elastic spheres, have furnished expressions for the laws of its elasticity, viscosity, conductivity for heat, diffusive power and other properties. For some of these laws we have experimental data of value in testing the validity of these deductions and assumptions. Next to the elasticity, perhaps the phenomena of the viscosity of gases are best adapted to investigation.[1]

Thesis supervisor: Edward C. Pickering
Title: Professor of Physics

[1] Text from Holman (1876): doi:10.2307/25138434.

Acknowledgments

Write your acknowledgments here.

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Chapter 1

Introduction

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1.1 A section discussing the first issue: J/ψ

We begin with some ideas from the literature [8,9].

$$\frac{\partial}{\partial t}\left[\rho\left(e+|\vec{u}|^2/2\right)\right]+\nabla\cdot\left[\rho\left(h+|\vec{u}|^2/2\right)\vec{u}\right] = -\nabla\cdot\vec{q}+\rho\vec{u}\cdot\vec{g}+\frac{\partial}{\partial x_j}\left(d_{ji}u_i\right) \tag{1.1}$$

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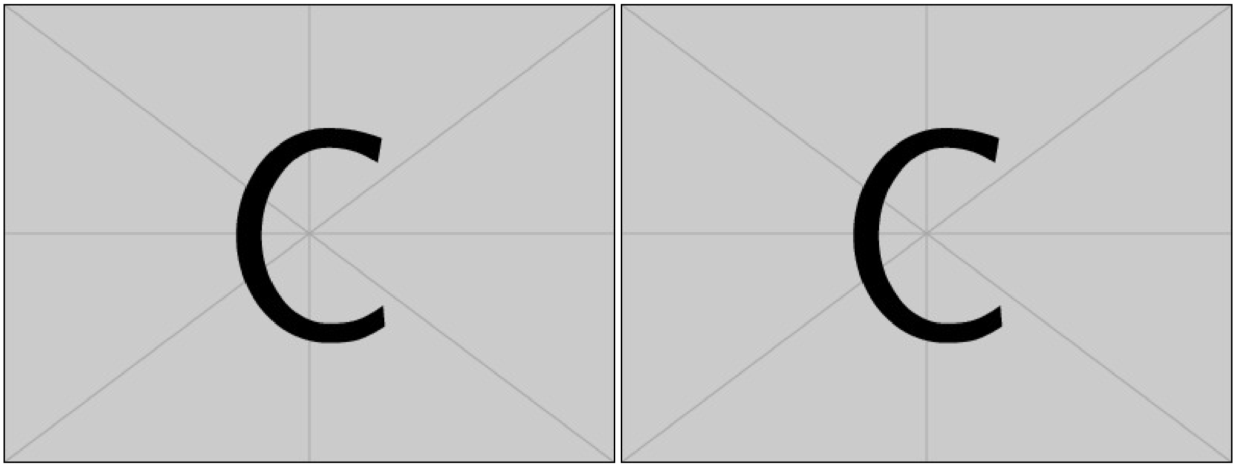

(a) Postremo aliquos futuros suspicor, qui me ad alias litteras vocent, genus hoc scribendi.

(b) Second subfigure.

Figure 1.1: A figure with two subfigures.

quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.

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Quisque ullamcorper placerat ipsum. Cras nibh. Morbi vel justo vitae lacus tincidunt ultrices. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. In hac habitasse platea dictumst. Integer tempus convallis augue. Etiam facilisis. Nunc elementum fermentum wisi. Aenean placerat. Ut imperdiet, enim sed gravida sollicitudin, felis odio placerat quam, ac pulvinar elit purus eget enim. Nunc vitae tortor. Proin tempus nibh sit amet nisl. Vivamus quis tortor vitae risus porta vehicula.

1.1.1 Subsection eqn. (1.2)

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A subsubsection

Sed commodo posuere pede. Mauris ut est. Ut quis purus. Sed ac odio. Sed vehicula hendrerit sem. Duis non odio. Morbi ut dui. Sed accumsan risus eget odio. In hac habitasse platea dictumst. Pellentesque non elit. Fusce sed justo eu urna porta tincidunt. Mauris felis odio, sollicitudin sed, volutpat a, ornare ac, erat. Morbi quis dolor. Donec pellentesque, erat ac sagittis semper, nunc dui lobortis purus, quis congue purus metus ultricies tellus. Proin et quam. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos hymenaeos. Praesent sapien turpis, fermentum vel, eleifend faucibus, vehicula eu, lacus.

$$L(\mathbf{A}) = \begin{pmatrix} \frac{\varphi}{(\varphi_1, \varepsilon_1)} & 0 & \dots & \dots & \dots & 0 \\ \frac{\varphi k_{2,1}}{(\varphi_2, \varepsilon_1)} & \frac{\varphi}{(\varphi_2, \varepsilon_2)} & 0 & \dots & \dots & 0 \\ \frac{\varphi k_{3,1}}{(\varphi_3, \varepsilon_1)} & \frac{\varphi k_{3,2}}{(\varphi_3, \varepsilon_2)} & \frac{\varphi}{(\varphi_3, \varepsilon_3)} & 0 & \dots & 0 \\ \vdots & \vdots & & \ddots & & \vdots \\ \frac{\varphi k_{n-1,1}}{(\varphi_{n-1}, \varepsilon_1)} & \frac{\varphi k_{n-1,2}}{(\varphi_{n-1}, \varepsilon_2)} & \dots & \frac{\varphi k_{n-1,n-2}}{(\varphi_{n-1}, \varepsilon_{n-2})} & \frac{\varphi}{(\varphi_{n-1}, \varepsilon_{n-1})} & 0 \\ \frac{\varphi k_{n,1}}{(\varphi_n, \varepsilon_1)} & \frac{\varphi k_{n,2}}{(\varphi_n, \varepsilon_2)} & \dots & \dots & \frac{\varphi k_{n,n-1}}{(\varphi_n, \varepsilon_{n-1})} & \frac{\varphi}{(\varphi_n, \varepsilon_n)} \end{pmatrix} \tag{1.2}$$

1.2 Description our paradigm

Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Donec odio elit, dictum in, hendrerit sit amet, egestas sed, leo. Praesent feugiat sapien aliquet odio. Integer vitae justo. Aliquam vestibulum fringilla lorem. Sed neque lectus, consectetuer at, consectetuer sed, eleifend ac, lectus. Nulla facilisi. Pellentesque eget lectus. Proin eu metus. Sed porttitor. In hac habitasse platea dictumst. Suspendisse eu lectus. Ut mi mi, lacinia sit amet, placerat et, mollis vitae, dui. Sed ante tellus, tristique ut, iaculis eu, malesuada ac, dui. Mauris nibh leo, facilisis non, adipiscing quis, ultrices a, dui. No dissertation is complete without footnotes.[1,2,3]

[1] First footnote. $a_h = F_m$ See section 1.4.

[2] Another interesting detail.

[3] And another really important idea to have in mind [13–18].

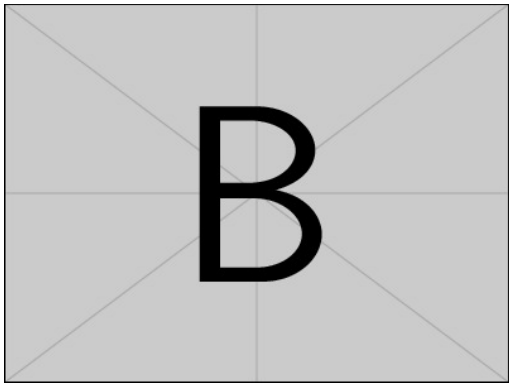

Figure 1.2: Caption text [1].

1.2.1 Conversion to a metaheuristic

Sed feugiat. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Ut pellentesque augue sed urna. Vestibulum diam eros, fringilla et, consectetuer eu, nonummy id, sapien. Nullam at lectus. In sagittis ultrices mauris. Curabitur malesuada erat sit amet massa. Fusce blandit. Aliquam erat volutpat. Aliquam euismod. Aenean vel lectus. Nunc imperdiet justo nec dolor.

Etiam euismod. Fusce facilisis lacinia dui. Suspendisse potenti. In mi erat, cursus id, nonummy sed, ullamcorper eget, sapien. Praesent pretium, magna in eleifend egestas, pede pede pretium lorem, quis consectetuer tortor sapien facilisis magna. Mauris quis magna varius nulla scelerisque imperdiet. Aliquam non quam. Aliquam porttitor quam a lacus. Praesent vel arcu ut tortor cursus volutpat. In vitae pede quis diam bibendum placerat. Fusce elementum convallis neque. Sed dolor orci, scelerisque ac, dapibus nec, ultricies ut, mi. Duis nec dui quis leo sagittis commodo. This concept is discussed further in section 1.4, and Refs. [19,20].

1.3 Other generalizations

Sed commodo posuere pede. Mauris ut est. Ut quis purus. Sed ac odio. Sed vehicula hendrerit sem. Duis non odio. Morbi ut dui. Sed accumsan risus eget odio. In hac habitasse platea dictumst. Pellentesque non elit. Fusce sed justo eu urna porta tincidunt. Mauris felis odio, sollicitudin sed, volutpat a, ornare ac, erat. Morbi quis dolor. Donec pellentesque, erat ac sagittis semper, nunc dui lobortis purus, quis congue purus metus ultricies tellus. Proin et quam. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos hymenaeos. Praesent sapien turpis, fermentum vel, eleifend faucibus, vehicula eu, lacus. And another citation, so that our sources will be unambiguous [21].

1.3.1 The most general case

Aliquam lectus. Vivamus leo. Quisque ornare tellus ullamcorper nulla. Mauris porttitor pharetra tortor. Sed fringilla justo sed mauris. Mauris tellus. Sed non leo. Nullam elementum, magna in cursus sodales, augue est scelerisque sapien, venenatis congue nulla arcu et pede. Ut suscipit enim vel sapien. Donec congue. Maecenas urna mi, suscipit in, placerat ut, vestibulum ut, massa. Fusce ultrices nulla et nisl. But, according to Lord Brouncker (1655),

$$\frac{4}{\pi} = 1 + \cfrac{1^2}{2 + \cfrac{3^2}{2 + \cfrac{5^2}{2 + \cfrac{7^2}{2 + \cfrac{9^2}{2 + \cdots}}}}} \tag{1.3}$$

1.4 Baroclinic generation of vorticity

Substitution of the particle acceleration and application Stokes theorem leads to the *Kelvin-Bjerknes circulation theorem*, for $\rho \neq \mathrm{fn}(p)$:

$$\begin{aligned}
\frac{d\Gamma}{dt} &= \frac{d}{dt}\int_C \mathbf{u}\cdot d\mathbf{r} && (1.4)\\
&= \int_C \frac{D\mathbf{u}}{Dt}\cdot d\mathbf{r} + \underbrace{\int_C \mathbf{u}\cdot d\left(\frac{d\mathbf{r}}{dt}\right)}_{=0} && (1.5)\\
&= \iint_S \nabla\times\frac{D\mathbf{u}}{Dt}\cdot d\mathbf{A} && (1.6)\\
&= \iint_S \nabla p\times\nabla\left(\frac{1}{\rho}\right)\cdot d\mathbf{A} && (1.7)
\end{aligned}$$

Baroclinic generation of vorticity accounts for the sea breeze and various other atmospheric currents in which temperature, rather than pressure, creates density gradients. Further, this phenomenon accounts for ocean currents in straits joining more and less saline seas, with surface currents flowing from the fresher to the saltier water and with bottom current going oppositely.

Weak shock wave. A weak shock wave travelling in a perfect gas is described by the following Burgers equation derived from approximations due to G. I. Taylor (1910)

$$u_t + \left(a_0 + \tfrac{1}{2}(\gamma+1)u\right)u_x = \frac{\delta}{2}u_{xx} \tag{1.8}$$

Here, γ is the specific heat capacity ratio, δ is the diffusivity of sound, u_1 is the downstream speed, $u_2 < u_1$ is the upstream speed, and a_0 is the undisturbed sound speed. The travelling-wave, or Taylor shock, solution of this equation is

$$\frac{u-u_2}{u_1-u_2} = \frac{1}{2}\left[1-\tanh\left(\frac{\gamma+1}{4\delta}(u_1-u_2)(x-ct)\right)\right] \tag{1.9}$$

where $c = a_0 + (\gamma+1)(u_1+u_2)/4$. This solution exists for a compressive wave, $u_1 > u_2$.

Table 1.1: The error function and complementary error function

x	$\text{erf}(x)$	$\text{erfc}(x)$	x	$\text{erf}(x)$	$\text{erfc}(x)$
0.00	0.00000	1.00000	1.10	0.88021	0.11980
0.05	0.05637	0.94363	1.20	0.91031	0.08969
0.10	0.11246	0.88754	1.30	0.93401	0.06599
0.15	0.16800	0.83200	1.40	0.95229	0.04771
0.20	0.22270	0.77730	1.50	0.96611	0.03389
0.30	0.32863	0.67137	1.60	0.97635	0.02365
0.40	0.42839	0.57161	1.70	0.98379	0.01621
0.50	0.52050	0.47950	1.80	0.98909	0.01091
0.60	0.60386	0.39614	1.8214	0.99000	0.01000
0.70	0.67780	0.32220	1.90	0.99279	0.00721
0.80	0.74210	0.25790	2.00	0.99532	0.00468
0.90	0.79691	0.20309	2.50	0.99959	0.00041
1.00	0.84270	0.15730	3.00	0.99998	0.00002

Table 1.2: Table with more complicated columns: $\Delta T_m = \delta\theta/\Theta_0$

Experiment	u [m/s]	T [°C]
The first test we ran this morning	124.3	68.3
The second test we ran this morning	82.50	103.46
Our competitor's test	72.321	141.384

1.5 Summary

Etiam ac leo a risus tristique nonummy. Donec dignissim tincidunt nulla. Vestibulum rhoncus molestie odio. Sed lobortis, justo et pretium lobortis, mauris turpis condimentum augue, nec ultricies nibh arcu pretium enim. Nunc purus neque, placerat id, imperdiet sed, pellentesque nec, nisl. Vestibulum imperdiet neque non sem accumsan laoreet. In hac habitasse platea dictumst. Etiam condimentum facilisis libero. Suspendisse in elit quis nisl aliquam dapibus. Pellentesque auctor sapien. Sed egestas sapien nec lectus. Pellentesque vel dui vel neque bibendum viverra. Aliquam porttitor nisl nec pede. Proin mattis libero vel turpis. Donec rutrum mauris et libero. Proin euismod porta felis. Nam lobortis, metus quis elementum commodo, nunc lectus elementum mauris, eget vulputate ligula tellus eu neque. Vivamus eu dolor.

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Table 1.3: Brinkman number at which liquid dissipation makes $< 5\%$ or $< 1\%$ contribution to θ_b or q_w from the Graetz solution at the points where θ_b equals 0.05, 0.02, or 0.01.

			|Br|			
θ_b	x^+	$q_w D_h / k\Delta T$	5% of θ_b	1% of θ_b	5% of q_w	1% of q_w
0.05	0.09621	0.3770	3.646×10^{-3}	7.292×10^{-4}	1.571×10^{-3}	3.142×10^{-4}
0.02	0.1266	0.1508	1.458×10^{-3}	2.917×10^{-4}	6.824×10^{-4}	1.257×10^{-4}
0.01	0.1496	0.07541	7.292×10^{-4}	1.458×10^{-4}	3.142×10^{-4}	6.284×10^{-5}

in, cursus faucibus, augue.

Nulla mattis luctus nulla. Duis commodo velit at leo. Aliquam vulputate magna et leo. Nam vestibulum ullamcorper leo. Vestibulum condimentum rutrum mauris. Donec id mauris. Morbi molestie justo et pede. Vivamus eget turpis sed nisl cursus tempor. Curabitur mollis sapien condimentum nunc. In wisi nisl, malesuada at, dignissim sit amet, lobortis in, odio. Aenean consequat arcu a ante. Pellentesque porta elit sit amet orci. Etiam at turpis nec elit ultricies imperdiet. Nulla facilisi. In hac habitasse platea dictumst. Suspendisse viverra aliquam risus. Nullam pede justo, molestie nonummy, scelerisque eu, facilisis vel, arcu.

Nomenclature for Chapter 1

Roman letters

- C material curve
- $\mathbf{r}$ material position [m]
- $\mathbf{u}$ velocity [m s^{-1}]

Greek letters

- Γ circulation [m^2 s^{-1}]
- ρ mass density [kg m^{-3}]
- $\boldsymbol{\omega}$ vorticity [s^{-1}]

Appendix A

Multipage and sideways tables

A.1 A multipage table of numbers

This example uses the `longtable` package. The values in the table are the one-term coefficients for one-dimensional, transient heat conduction, $\theta = A_1 f_1 \exp(-\lambda_1^2 \mathrm{Fo})$, $\overline{\theta} = D_1 \exp(-\lambda_1^2 \mathrm{Fo})$.

Table A.1: One-term coefficients for one-dimensional heat conduction with a convective boundary condition. Data follow H. D. Baehr and K. Stephan [2].

Bi	*Plate*			*Cylinder*			*Sphere*		
	λ_1	A_1	D_1	λ_1	A_1	D_1	λ_1	A_1	D_1
0.01	0.09983	1.0017	1.0000	0.14124	1.0025	1.0000	0.17303	1.0030	1.0000
0.02	0.14095	1.0033	1.0000	0.19950	1.0050	1.0000	0.24446	1.0060	1.0000
0.03	0.17234	1.0049	1.0000	0.24403	1.0075	1.0000	0.29910	1.0090	1.0000
0.04	0.19868	1.0066	1.0000	0.28143	1.0099	1.0000	0.34503	1.0120	1.0000
0.05	0.22176	1.0082	0.9999	0.31426	1.0124	0.9999	0.38537	1.0150	1.0000
0.06	0.24253	1.0098	0.9999	0.34383	1.0148	0.9999	0.42173	1.0179	0.9999
0.07	0.26153	1.0114	0.9999	0.37092	1.0173	0.9999	0.45506	1.0209	0.9999
0.08	0.27913	1.0130	0.9999	0.39603	1.0197	0.9999	0.48600	1.0239	0.9999
0.09	0.29557	1.0145	0.9998	0.41954	1.0222	0.9998	0.51497	1.0268	0.9999
0.10	0.31105	1.0161	0.9998	0.44168	1.0246	0.9998	0.54228	1.0298	0.9998
0.15	0.37788	1.0237	0.9995	0.53761	1.0365	0.9995	0.66086	1.0445	0.9996
0.20	0.43284	1.0311	0.9992	0.61697	1.0483	0.9992	0.75931	1.0592	0.9993
0.25	0.48009	1.0382	0.9988	0.68559	1.0598	0.9988	0.84473	1.0737	0.9990
0.30	0.52179	1.0450	0.9983	0.74646	1.0712	0.9983	0.92079	1.0880	0.9985
0.40	0.59324	1.0580	0.9971	0.85158	1.0931	0.9970	1.05279	1.1164	0.9974
0.50	0.65327	1.0701	0.9956	0.94077	1.1143	0.9954	1.16556	1.1441	0.9960
0.60	0.70507	1.0814	0.9940	1.01844	1.1345	0.9936	1.26440	1.1713	0.9944
0.70	0.75056	1.0918	0.9922	1.08725	1.1539	0.9916	1.35252	1.1978	0.9925
0.80	0.79103	1.1016	0.9903	1.14897	1.1724	0.9893	1.43203	1.2236	0.9904
0.90	0.82740	1.1107	0.9882	1.20484	1.1902	0.9869	1.50442	1.2488	0.9880
1.00	0.86033	1.1191	0.9861	1.25578	1.2071	0.9843	1.57080	1.2732	0.9855
1.10	0.89035	1.1270	0.9839	1.30251	1.2232	0.9815	1.63199	1.2970	0.9828
1.20	0.91785	1.1344	0.9817	1.34558	1.2387	0.9787	1.68868	1.3201	0.9800
1.30	0.94316	1.1412	0.9794	1.38543	1.2533	0.9757	1.74140	1.3424	0.9770
1.40	0.96655	1.1477	0.9771	1.42246	1.2673	0.9727	1.79058	1.3640	0.9739

Table A.1: (continued)

Bi	*Plate*			*Cylinder*			*Sphere*		
	λ_1	A_1	D_1	λ_1	A_1	D_1	λ_1	A_1	D_1
1.50	0.98824	1.1537	0.9748	1.45695	1.2807	0.9696	1.83660	1.3850	0.9707
1.60	1.00842	1.1593	0.9726	1.48917	1.2934	0.9665	1.87976	1.4052	0.9674
1.70	1.02725	1.1645	0.9703	1.51936	1.3055	0.9633	1.92035	1.4247	0.9640
1.80	1.04486	1.1695	0.9680	1.54769	1.3170	0.9601	1.95857	1.4436	0.9605
1.90	1.06136	1.1741	0.9658	1.57434	1.3279	0.9569	1.99465	1.4618	0.9570
2.00	1.07687	1.1785	0.9635	1.59945	1.3384	0.9537	2.02876	1.4793	0.9534
2.20	1.10524	1.1864	0.9592	1.64557	1.3578	0.9472	2.09166	1.5125	0.9462
2.40	1.13056	1.1934	0.9549	1.68691	1.3754	0.9408	2.14834	1.5433	0.9389
2.60	1.15330	1.1997	0.9509	1.72418	1.3914	0.9345	2.19967	1.5718	0.9316
2.80	1.17383	1.2052	0.9469	1.75794	1.4059	0.9284	2.24633	1.5982	0.9243
3.00	1.19246	1.2102	0.9431	1.78866	1.4191	0.9224	2.28893	1.6227	0.9171
3.50	1.23227	1.2206	0.9343	1.85449	1.4473	0.9081	2.38064	1.6761	0.8995
4.00	1.26459	1.2287	0.9264	1.90808	1.4698	0.8950	2.45564	1.7202	0.8830
4.50	1.29134	1.2351	0.9193	1.95248	1.4880	0.8830	2.51795	1.7567	0.8675
5.00	1.31384	1.2402	0.9130	1.98981	1.5029	0.8721	2.57043	1.7870	0.8533
6.00	1.34955	1.2479	0.9021	2.04901	1.5253	0.8532	2.65366	1.8338	0.8281
7.00	1.37662	1.2532	0.8932	2.09373	1.5411	0.8375	2.71646	1.8673	0.8069
8.00	1.39782	1.2570	0.8858	2.12864	1.5526	0.8244	2.76536	1.8920	0.7889
9.00	1.41487	1.2598	0.8796	2.15661	1.5611	0.8133	2.80443	1.9106	0.7737
10.00	1.42887	1.2620	0.8743	2.17950	1.5677	0.8039	2.83630	1.9249	0.7607
12.00	1.45050	1.2650	0.8658	2.21468	1.5769	0.7887	2.88509	1.9450	0.7397
14.00	1.46643	1.2669	0.8592	2.24044	1.5828	0.7770	2.92060	1.9581	0.7236
16.00	1.47864	1.2683	0.8541	2.26008	1.5869	0.7678	2.94756	1.9670	0.7109
18.00	1.48830	1.2692	0.8499	2.27556	1.5898	0.7603	2.96871	1.9734	0.7007
20.00	1.49613	1.2699	0.8464	2.28805	1.5919	0.7542	2.98572	1.9781	0.6922
25.00	1.51045	1.2710	0.8400	2.31080	1.5954	0.7427	3.01656	1.9856	0.6766
30.00	1.52017	1.2717	0.8355	2.32614	1.5973	0.7348	3.03724	1.9898	0.6658
35.00	1.52719	1.2721	0.8322	2.33719	1.5985	0.7290	3.05207	1.9924	0.6579
40.00	1.53250	1.2723	0.8296	2.34552	1.5993	0.7246	3.06321	1.9942	0.6519
50.00	1.54001	1.2727	0.8260	2.35724	1.6002	0.7183	3.07884	1.9962	0.6434
60.00	1.54505	1.2728	0.8235	2.36510	1.6007	0.7140	3.08928	1.9974	0.6376
80.00	1.55141	1.2730	0.8204	2.37496	1.6013	0.7085	3.10234	1.9985	0.6303
100.00	1.55525	1.2731	0.8185	2.38090	1.6015	0.7052	3.11019	1.9990	0.6259
200.00	1.56298	1.2732	0.8146	2.39283	1.6019	0.6985	3.12589	1.9998	0.6170
∞	1.57080	1.2732	0.8106	2.40483	1.6020	0.6917	3.14159	2.0000	0.6079

A.2 A sideways table

Table A.2 uses the `sidewaystable*` environment from the `rotating` package.

Table A.2: Data sets for local heat transfer coefficients in liquids. Boundary condition (B.C.) and r.m.s. freestream turbulence (u'_r/u_∞) are as indicated.

Authors	**B.C.**	**Range of Re$_x$**	u'_r/u_∞	**Comments**
Data for Water				
Hollingsworth, 1989	UWT	$\approx 5 \times 10^5$	$0.5 - 1.5\%$	Data for $\mathrm{Pr} = 5.93 \pm 0.13$ with 4 K b.l. temperature difference. Boundary layer tripped ahead of plate, with a substantial unheated section; two locations measured in uncurved channel. Plate temperature controlled by rows of electrically heated strips. Data analysis focuses on enthalpy thickness Reynolds number. Data were used to calibrate a numerical model, leading to a correlation: $\mathrm{St} = 0.02426\ \mathrm{Re}_x^{-n}\mathrm{Pr}^{-0.895}$ for $n = 0.1879\ \mathrm{Pr}^{-0.18}$ for $0.7 \leqslant \mathrm{Pr} \leqslant 8$. Measurement uncertainties (2σ): $\mathrm{St} \pm 3.3\%$; $\mathrm{Re}_x \pm 1.6\%$.
Žukauskas & Šlančiauskas, 1987 summary	UHF	$4.0 \times 10^3 - 4.4\times10^6$	$\sim 1.2\%$	Data for Prandtl numbers of 2.95, 5.4, and 6.6. Stated uncertainty in heat transfer is up to $\pm10\%$, being largest for higher fluxes. Background turbulence measured at inlet to test section. For Pr = 5.4: Run 17, ΔT is 7–8 K for the turbulent and transitional data, with the laminar points rising to 17 K; Run 18, $6.5 \leqslant \Delta T \leqslant 21.4$ K; Table 25, ΔT is 7–8 K for the turbulent and transitional data, again rising to 15–17 K in the laminar region. For Pr = 6.6: Run 7, $1.3 \leqslant \Delta T \leqslant 4.2$ K; Run 8, $6.5 \leqslant \Delta T \leqslant 21.4$ K. For Pr = 2.95: $5.4 \leqslant \Delta T \leqslant 8.6$ K.
Data for higher Pr				
Žukauskas & Šlančiauskas, 1987 summary	UHF	$1.2 \times 10^4 - 7.7\times10^5$	$\sim 0.7\%$	Data for “transformer oil” at several Prandtl numbers (55–257). Uncertainty in heat transfer stated as $\pm5\%$. Background turbulence measured at inlet to test section. Much of this data has significant property variation across the boundary layer. (An additional data set is given for cooling hot glycerin at higher Pr, but with even stronger property variations.)

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